Indeterminate numerals and their alternatives *

Curt Anderson
Heinrich-Heine-Universität Düsseldorf

1 Introduction

Approximation in English can be expressed in various ways. For instance, the adverbials *almost* and *approximately* are some ways of expressing that a numerical expression should be construed approximately, that is, to express uncertainty regarding the precise number that expression should denote. Prepositions provide another way of expressing approximation.

(1) a. around ten people
b. between ten and twenty people
c. close to ten people

In this paper, I look at a type of approximative construction in English involving numerals and an indefinite determiner, as in (2) below. With these numerals, which I call indeterminate numerals, *some* appears post-numerally, affixed to the preceding numeral. The interpretation in these examples is one where the indeterminate numeral expresses a range of possible numbers, but where the speaker doesn't know the precise number that satisfies the existential claim expressed by the sentence, as observed by Anderson (2015, 2016). These numerals are theoretically interesting due to their reliance on the epistemic indefinite *some*. This sets it apart syntactically from other instances of approximation, in that the element that is expressing approximation is not an adverbial or a preposition.

(2) a. Twenty-some people arrived.
b. His forty-some years of experience were devoted to human resources.
c. I could have it entirely full of small icons and fit a hundred some icons on one screen.
d. More than half of the expenditure of eighty-some thousand dollars is for soft costs.

*I thank Marcin Morzycki, Peter Hallman, Keir Moulton, Ai Taniguchi, Adam Gobeski, Jon Ander Mendia, and two anonymous reviewers for their discussion and comments. I was funded by DFG Collaborative Research Center 991, projects B09 and C10, while writing this paper. Naturally, all errors are my own.

However, these numerals are restricted syntactically. *Some* is not a simple ad-numeral affix, but seems to be integrated within the syntactic structure of the numeral; only numerals which support additive composition can support *some*.

(3) a. *five-some
b. *ten-some
c. *fifteen-some

Interestingly, these numerals are also curious in that they simultaneously express both an upper-bounded and lower-bounded meaning. *Twenty-some*, for instance, expresses that any number between 20 and 30 is a possibility. In some ways, this makes them superficially similar to modified numerals such as *at least ten* and *not more than twenty*, but also different from them in having this sort of two-sided meaning.

Discussion of the semantics of numerals has often gone hand-in-hand with that of canonical quantificational determiners like *every* and *most*, with the question being of how and whether cardinal numerals differ from quantificational determiners in their type-theoretic properties. Possibilities include treating numerals as quantificational determiners (type $\langle et, \langle et, t \rangle \rangle$; e.g., Barwise & Cooper 1981, Hofweber 2005), as degree quantifiers (type $\langle dt, t \rangle$; see Kennedy 2015), as cardinality predicates (type $\langle e, t \rangle$; e.g., Landman 2003, Rothstein 2013) or predicate modifiers (type $\langle et, et \rangle$; e.g. Ionin & Matushansky 2006), and as degree-denoting terms (type d) with additional functional machinery mediating between the noun phrase and the numeral (e.g., Solt 2015).[1] Likewise, modified numerals like *at least sixty* and *no more than fifteen*, which indeterminate numerals bear some resemblance to, also have generated discussion as to their logical form, particularly about whether they are quantificational determiners (Barwise & Cooper 1981) or degree quantifiers (Nouwen 2010, Kennedy 2015). Indeed, some surveys of quantification devote space to discussion of both modified and unmodified numerals (for instance, Szabolcsi 2010: chapters 9 and 10).

These indeterminate numerals also bring into relief the way that degrees have been implicated in many areas of natural language meaning. That degree constructions such as comparatives and superlatives are degree quantifiers seems to now be the standard view (see Morzycki's (2016) textbook for discussion of these and other degree constructions, for instance), showing that there exist parallels between quantification over individuals and degrees. Degrees have also been implicated in many phenomena related to gradability and scalarity across lexical categories (not just gradable adjectives), again showing that degree variables and reference to degrees is as pervasive as that of individuals. The present chapter contributes to

[1] See Geurts 2006 for additional discussion of these issues.

this view by showing that at least one indefinite determiner, *some*, can also quantify over degrees. Having degrees represented in the determiner system nudges this parallelism between degrees and individuals even further, and (as noted by Hallman (this volume)), reflects the way degrees have steadily worked their way into systems once thought to be reserved for operations over sets of individuals.

With the big picture in mind, I return to the matter at hand: indeterminate numerals. My analysis of indeterminate numerals makes use of a few key ingredients. First, I claim that the *some* in indeterminate numerals is the epistemic indefinite *some*. As an epistemic indefinite, *some* signals uncertainty regarding the precise referent that satisfies a description. *Some* has an identical flavor with numerals, in that it fails to commit the speaker to knowledge of the particular number that satisfies the numerical description. Second, the ignorance is derived from the properties of *some* itself. I take *some* to impose a requirement that there exists a non-singleton set of alternatives (in this case, numerical alternatives), with ignorance derived as an implicature. These alternatives are part of the compositional machinery of the sentence, following the framework developed by Kratzer & Shimoyama (2002). Third, I make a proposal for the syntax of indeterminate numerals, arguing that the numeral itself forms a constituent (to the exclusion of the NP, contra Ionin & Matushansky (2006)).

Looking at indeterminate numerals expands on our understanding of these groups of expressions and how quantificational elements like indefinite determiners interact with degrees denoted in domains other than the adjective phrase. Indeterminate numerals show an interaction of degree and quantification due to how properties of the indefinite determiner *some* (particularly, its ability to force the generation of multiple Hamblin alternatives) interact with the numeral to produce quantification over sets of alternatives that vary by degree. Additionally, looking at these numerals gives us insight into the division of labor between asserted and implicated meanings in complex numerals: I show that numerals modified by *some* have an assert lower-bound, but that the upper-bound is generated via implicature.

I structure this chapter in the following way. First, in section 2, I discuss additional background data on English indeterminate numerals as well as link them to the broader category of epistemic indefinites. Next, in section 3, I give a syntax for numerals in general that will be necessary to have for the analysis of indeterminate numerals. Section 4 lays out background on the alternative semantics used in the rest of the analysis in the paper. Section 5 develops an account of the semantics of ordinary numerals, while sections 6 and 7 develop the analysis of indeterminate numerals.

2 Background data

2.1 Expanding on the phenomenon

Modified numerals such as *at least 10* and *not more than 20* have bounded interpretations, either lower-bounded (like with *at least*) or upper-bounded (like with *not more than*). What sets indeterminate numerals apart from many other cases of modified numerals is that they are both lower-bounded and upper-bounded. For instance, the numerals in the examples in (2) are associated with the intervals as in (4). The salient fact about this interval is that its lower bound starts at the modified numeral, and has an upper-bound as determined by keeping the base of the modified numeral and increasing the multiplier by one unit. For instance, *twenty* is represented as 2×10, so by keeping the base 10 constant and increasing the multiplier from 2 to 3, we arrive at the upper-bound for *twenty-some*. Likewise, *hundred* is represented as 1×100, so the upper-bound of *hundred-some* is represented as 2×100.

(4) a. twenty-some $\rightsquigarrow (20, 30)$
b. forty-some $\rightsquigarrow (40, 50)$
c. hundred-some $\rightsquigarrow (100, 200)$

This makes indeterminate numerals different than approximators, such as *around* and *about*. Although they seem similar in that they involve a number that is close to what is being modified, *around* implicates a halo of numbers centered around the modified numeral (for instance, something like $[18 - 22]$ in (5)), while the interval for the indeterminate numeral is bounded on the lower end by the number denoted by the numeral.

(5) I saw around twenty dogs during my walk today.
(= I saw between 18 and 22 days during my walk today.)

It's tricky to show that there is a particular number that sets the lower bound, due to the epistemic requirement that the speaker do not know the precise number that satisfies the claim. But, if we pair an utterance with a fact about the world that the speaker learns later on, we can show that the utterance was either true or false. When we pair (6) with (7a), where the fact of the matter is that there was a number of dogs incompatible with *twenty-some*, namely 19 dogs, the sentence is judged false. However, if (6) is paired with (7b), where the fact is that there were actually 23 dogs the speaker saw, then the utterance is judged to be true. This shows that the utterance really is lower-bounded by the numeral that is being modified.

(6) I saw twenty-some dogs during my walk today.

(7) a. *Speaker later learns he saw only 19 dogs:*
(6) is judged to have been false.
b. *Speaker later learns he saw 23 dogs:*
(6) is judged to have been true.

Moreover, the lower-bound is at the modified numeral, but does not include the denotation of said numeral. The examples in (8) and (9) are quite marginal, providing evidence that the lower-bound for e.g. *twenty-some* does not include the number 20, but rather starts at 21.

(8) ??I saw twenty-some dogs today, namely exactly twenty.

(9) *(Situation: John ate exactly twenty cookies.)*
??John ate twenty-some cookies.

Returning to the question of how and where *some* is licensed, what we observe is that indeterminate numerals in English are only possible if the modified numeral is one that can combine additively with another numeral. When the numeral cannot combine additively with another numeral, as is the case with *one* through *nineteen*, an indeterminate numeral is impossible.

(10) a. *ten-some
b. *five-some

(11) a. *ten-five (expected: 15)
b. *five-one (expected: 6)

Moreover, *some* does not have to occur after the entire phrase corresponding to the numeral. If a smaller constituent can combine additively with another numeral, *some* can appear in that position, as in (12).

(12) More than half of the expenditure of eighty-some thousand dollars is for soft costs.

A brief discussion of additivity is in order. What I mean by additivity is that the meaning of certain numerals is derived via addition of degrees. In the case of the numeral *twenty-five*, for instance, the meaning of *twenty-five* is derived via addition of the meanings for *twenty* and *five*, e.g. $20+5$. Not all numerals support additive composition with a constituent to their right; *twenty* can combine additively with another numeral since the constituent immediately to its right (informally speaking) is semantically interpreted as being added to the meaning of *twenty*, but this is not a property of *ten*, since *ten* does not additive combine with a numeral to its right (e.g., **ten five* for 15). This contrasts with multiplicative composition, where some numerals combine with another numeral via multiplication of that numeral plus a base, such as with *ten thousand* or *two million*. These observations regarding the compositionality of the numeral system are not new, and go back at least to Hurford (1975), who provides an early phrase structural account of numerals in a variety of languages.

Perhaps unsurprisingly, these same facts are also found with measure phrases. This shows that we are looking at a phenomenon that is quite generally related to measurement and degree, and not only to counting constructions within the DP.

(13) a. The Empire State Building is 440-some meters tall.
b. He is 20-some years old.

An understanding of the position of *some* in the syntax of the numeral, the lower and upper-bound of the scale, and the ignorance in the construction form the basic desideratum of an account of English indeterminate numerals.

2.2 Indeterminate numerals as epistemic indefinites

The driving idea behind the analysis is that indeterminate numerals like *twenty-some* are a variety of epistemic indefinite. Epistemic indefinites are indefinites that convey ignorance on the part of the speaker as to the particular referent of some nominal expression. They are quite robustly attested cross-linguistically with examples in English (*some*), German (*irgendein*), Spanish (*algún*), Romanian (*vreun*), Hungarian (*vagy*), and Japanese (the WH-*ka* series of pronouns).[2]

Rather than express ignorance as to the identify of an individual, however, what the indeterminate numeral does is express ignorance as to the precise number that satisfies a description. In other words, while ordinary epistemic indefinites contribute uncertainty as to the witnessing individual for a linguistic description, indeterminate numerals contribute uncertainty with respect to the witnessing number. To motivate this view that indeterminate numerals really are epistemic indefinites, we have to first compare their properties with another well-known epistemic indefinite. The epistemic indefinites that I compare the indeterminacy-building element in indeterminate numerals to are *some* in its canonical determiner use, as well as Spanish *algún*.

Some implicates that the speaker doesn't know the precise identity of the person being referred to. The examples in (14) and (15) below (attributable to Strawson (1974)) demonstrate this contrast with *a* and *some*. While person B cannot ask the question about who was shot in the exchange in (14), due to person A having used *some*, this is allowed in (15), due to the indefinite *a* being compatible with knowledge on the part of the speaker.

[2]See Haspelmath 1997 and Alonso-Ovalle & Menéndez-Benito 2013 for overviews.

(14) A: Some cabinet minister has been shot!
B: #Who?

(15) A: A cabinet minister has been shot!
B: Who?

Comparing the behavior of the indeterminate numeral to *some*, we can see that it requires the same expression of ignorance. This is illustrated in (16), where someone cannot follow-up an utterance that uses an indeterminate numeral by asking for an exact quantity.[3]

(16) A: Twenty-some students are taking my class this semester
B: #How many?

Alonso-Ovalle & Menéndez-Benito (2010) note that the ignorance inference with *algún* can be reinforced with other linguistic material. This sets it apart from presuppositional content and asserted content, which cannot be reinforced, due to being entailed. Thus, the fact that the ignorance inference can be reinforced suggests that the inference is not entailed, but is rather an implicature. (17) demonstrates this with *algún*, where the clause following *pero* 'but' reinforces the ignorance expressed in the first clause. (18) demonstrates an equivalent sentence in English, where the epistemic indefinite determiner *some* is used.

(17) *María sale con algún estudiante del departamento de lingüística*,
María goes out with ALGUN student of the department of linguistics,
pero no sé con quién
but not I know with whom.
'María is dating some student in the linguistics department, but I don't know who.' (Alonso-Ovalle & Menéndez-Benito 2010: (45d))

(18) Mary is dating some student in the linguistics department, but I don't know who.

Likewise, the expression of ignorance in the indeterminate numeral can be reinforced, drawing an additional parallel between known epistemic indefinites like *some* and *algún* on one hand, and indeterminate numerals.

[3] Keir Moulton (p.c.) suggests that *precisely how many* is a better question for B to follow-up with than *how many*. My own judgements here aren't very firm, but I think that *precisely how many* can be acceptable if the implicature generated by *some* is not ignorance in this case, but rather relevance or indifference. It's probably generally the case that the implicature that can be generated by *some* is not just ignorance, but a variety of implicatures related to being unable or unwilling to name an individual or degree. That observation that ignorance is one member of a family of inferences is a point made by Condoravdi (2015) for *wh-ever* free relatives and Coppock (2016) for *at least* and *at most*.

(19) Mary cooked twenty-some pies, but I don't know exactly how many.[4]

Alonso-Ovalle & Menéndez-Benito (2010) argue that Spanish *algún* is compatible with partial ignorance. A speaker using *algún* is not committed to total ignorance regarding the witness of an existential claim (which individual(s) make the proposition true), merely that they cannot in principle narrow the domain of the indefinite to fewer than two choices. If *algún* required total ignorance, that all epistemic possibilities are available, examples such as the one in (20) would be malformed, due to restrictions being placed on the set of alternatives. As (20) shows, however, *algún* doesn't require that all possibilities be open, only that there be at least two. This also seems to hold for *some*, in that a similar example in English is also perfectly licit in the same scenario.

(20) SCENARIO: María, Juan, and Pedro are playing hide-and-seek in their country house. Juan is hiding. María and Pedro haven't started looking for Juan yet. Pedro believes that Juan is not hiding in the garden or in the barn: he is sure that Juan is inside the house. Furthermore, Pedro is sure that Juan is not in the bathroom or in the kitchen. As far as he knows, Juan could be in any of the other rooms in the house. Pedro utters:

Juan tiene que estar en alguna habitación de la casa.
Juan has to be in ALGUNA room of the house
'Juan must be in a room of the house.' (based on Alonso-Ovalle & Menéndez-Benito 2010: (14) & (15))

(21) Juan must be in some room in the house.

Mendia (2018) makes a similar observation for indeterminate numerals; these numerals are also compatible with partial ignorance regarding the witnessing number, as shown in (22). A score in basketball is usually two points, but a triple is worth three points; adding additional information about the manner of scoring in this way serves to narrow down the set of possibilities for how much Michael Jordan actually scored.

(22) That night Michael Jordan scored twenty-some points in triples. (Mendia 2018: (43))

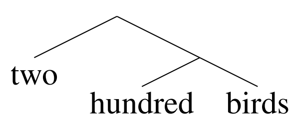
[4] This example gets worse or even unacceptable if *exactly* is left off: **Mary cooked twenty-some pies, but I don't know how many.* My suspicion is that this is due to a clash between *twenty-some* committing the speaker to some measure of pies (just not an exact measure), and *I don't know how many* committing the speaker to total ignorance. Since the speaker does assert he knows some number, just not the precise number, he can't go on to further assert he doesn't know the number at all.

To conclude this section, indeterminate numerals appear to pattern with other epistemic indefinites in that they also enforce an epistemic requirement on the speaker that the speaker not be able to make a precise claim as to the identity of the referent. With respect to numbers, this amounts to the speaker not being able to commit as to which particular number satisfies a description. This is similar to the behavior of *some* and *algún*. Moreover, like *algún* and *some*, the ignorance inference can be reinforced, making it pattern with implicatures rather than presuppositions and assertions. In the next sections, I'll develop an analysis of indeterminate numerals that uses insights from Alonso-Ovalle & Menéndez-Benito (2010)'s analysis of *algún*, and show how the ignorance inference can be generated as an implicature.

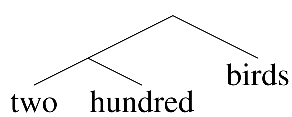
3 A syntax for numerals

3.1 Indeterminate numerals are in specifiers

The syntax of numerals has largely revolved around two competing approaches, what Danon (2012) calls the head-complement construction and the spec-head construction. Although the precise details regarding various proposals for these types of approaches vary, what primarily differentiates them is whether multiplicative numerals, such as *two hundred*, are constituents (to the exclusion of the NP they appear along with) or are represented hierarchically along the spine of the noun phrase. The possibilities are schematically represented in (23) and (24). In (23), the head-complement approach, the numerals are located along the spine of the tree, with the base numeral *hundred* taking the NP as its complement. This contrasts with the spec-head approach in (24), where the numeral itself is a constituent, to the exclusion of the NP.

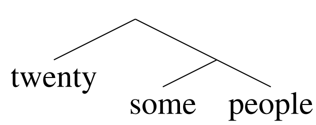
(23) 

(head-complement)

(24) 

(spec-head)

The facts regarding indeterminate numerals suggest that the appropriate structure in their case is a structure along the lines of (24), where the numeral (including *some*) form a constituent. The argument is as follows. First, suppose that the indeterminate numeral structure is as in (25), where *some* has as its complement the NP.

(25) 

(rejected analysis)

We observe that, in addition to *some*, English also allows for an equivalent structure containing *something*, as in (26).

(26) Similarly, Lauren and the other twenty-something people I observed had some structured group meals [...] (Google)

The ability of *something* to also appear in the indeterminate numeral construction is important, because it shows that *some* cannot be taking the NP as its complement. We are able to see that *something* cannot merge with an NP (possibly due to *some*'s complement already being filled by the noun *thing*). If the structure for indeterminate numerals were as in (25), with *something* in the same position as *some*, we would be forced to assume the existence of two separate instances of *something*: one that is incapable of appearing with an NP, and another that can have an NP as its complement, as in (28).

(27) some(*thing) people

(28) [twenty [something years]] old (rejected analysis)

Moreover, the existence of numeral internal *some* also points to *some* not taking the NP as a complement in examples such as *twenty-some people*.

(29) a. twenty-some thousand dollars
b. forty-some million Germans

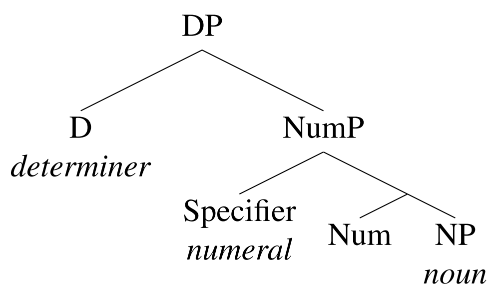
(30) [twenty [some [thousand dollars]]] (rejected analysis)

We might have analyzed these as *some* taking the numeral as a complement again, but as I point out in Anderson (2014), examples like *some twenty people* have a kind of approximative interpretation: at first glance, *some twenty* has a meaning similar to *approximately twenty*, allowing for *some twenty* to refer to numbers close to twenty. I proposed that this constructed via expanding the denotation of a numeral into its pragmatic halo (in the sense of Lasersohn (1999)) and then choosing from among this expanding denotation (see Sauerland & Stateva (2007) for an approach based on manipulating a granularity parameter and discussion of a variety of approximators, and also Stevens & Solt (2018), who argue that that examples like *some twenty people* have a different kind of semantics than *around twenty people*, *about twenty people*, and *approximately twenty people*). Thus, we would expect that the constituent *some thousand* get interpreted as an interval centered on 1000 (e.g., *some thousand* $\approx$ [990,1010]), and the entire indeterminate numeral have the interpretation "twenty counts of some thousand". But this is not what this means: *twenty-some thousand* is most naturally interpreted as a possible range of thousands, starting at twenty-one thousand and ending at twenty-nine thousand, as we would expect if *some* were a constituent with *twenty* and not *thousand*. Thus, the analysis in (30) must be rejected.

Based on these observations regarding *some*, I analyze *some* as forming a constituent with the numeral rather than the noun. This corresponds to the spec-head structure schematized in (24), rather than the complement structure in (23).

In order to link the numerosity denoted by the numeral up with the noun, I assume that a covert element is used in order to provide a degree argument. Some theories suppose a covert type-shift or adjective MANY/MUCH, which does the job of providing a degree argument via a measure function over individuals, returning their cardinality. Taking an approach closer to Solt (2015) and others who assume functional material in the DP mediating between numerals and the lexical NP, I syntacticize the measure function and place it in a functional head sister to the NP, Num (semantics to follow in later sections). The resulting structure thus looks like the following in (31).

(31)



3.2 ADD and complex numeral structure

In work on the syntax and semantics of numerals, Ionin & Matushansky (2006) argue that numerals largely have a structure where the numeral takes the noun as a complement, with parts of the numeral distributed along the spine of the tree rather than being in a specifier. (This corresponds to the head-complement analysis from the previous section.) Still, they need to address the fact that some numerals do have additional complexity that cannot be modeled in this way, such as with *two hundred twenty*. Ionin & Matushansky propose that these numerals are underlyingly coordinate structures.

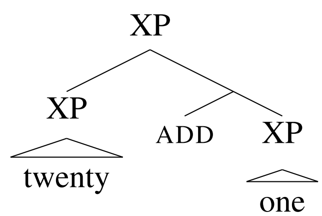
Direct evidence can be found with languages that overtly realize this conjunction; for instance, as shown in (32), both Spanish and German overtly realize an element meaning *and* in at least some numerals with additive complements, and English even optionally allows for *and* in some environments, as shown in (33). (See Ionin & Matushansky 2006 for additional details.)

(32) a. *fünfundzwanzig* (German)
five.and.twenty

b. *treinta y cinco* (Spanish)
thirty and five

(33) one hundred (and) one

I take complex additive numerals to have the structure in (34), which follows Ionin & Matushansky (2006) in the use of a covert coordination element. Departing slightly from Ionin & Matushansky, I call this ADD. This use of ADD builds on even earlier work by Hurford (1975), who develops an early account using phrase structure rules for how numerals are constructed in English and a selection of other languages. Hurford observes that syntactic positions are correlated with particular modes of composition (additive or multiplicative), and the use of a coordination-like element encoding the mode of composition essentially syntacticizes this earlier insight.[5]

(34)


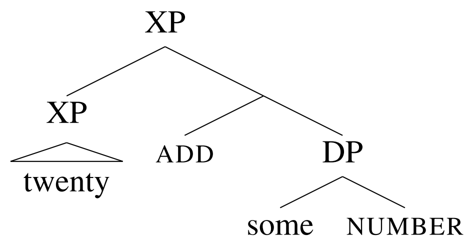
3.3 NUMBER as the complement to *some*

As demonstrated previously, English indeterminate numerals are only possible with additive numeral constructions. I analyze the *some* component of the construction as being like a numeral, albeit an indefinite numeral. In keeping with the pragmatic parallels between -*some* in the indeterminate numeral and the more canonical determiner *some*, I analyze *some* in this construction as a determiner as well, taking an NP complement.

Being in a complex numeral construction, *some* is combined with the numeral that it modifies via the ADD coordinator described in the previous section. The structure for indeterminate numerals is as in (35). I assume that the NP complement to *some* is a silent noun NUMBER. A covert nominal of this sort has been proposed to be at work in other phenomenon using numerals; Kayne (2005) proposes that *few* and *many* modify a silent noun NUMBER, while Zweig (2005) makes use of it in his syntax of numerals.

[5] Hurford (1975) predates many contemporary syntactic notions; additive and multiplicative composition are rules for semantic interpretation assigned to particular phrase structure rules, rather than read off of terminals in the tree as in our current tradition. Regardless, he clearly has the view that structure plays a role.

(35)



3.4 Blocking of illicit numerals

Before turning to the main analysis, I need to take a short detour to talk about how to rule out malformed numerals such as **twenty-eleven* (for *thirty-one*) and **forty-fifteen* (for *fifty-five*). The issue of how to constrain the compositionality of the numeral system has been vexing problem since at least Hurford (1975). The fundamental problem is that, while it is reasonably straightforward to describe the mathematical contribution of each individual component of a complex numeral, particular mathematically equivalent strings are ruled out; **twenty-eleven* and *thirty-one* should be able to name the same number, but only *thirty-one* seems to be a well-formed numeral in English.

One possibility that I speculate about in Anderson 2016 is the use of syntactic features corresponding to numerical bases (e.g., ones, tens, and hundreds). Additive numerals could use feature checking systems to ensure that the numerical base of their sister is smaller. Mendia (2018) independently develops a similar strategy, with the intention of generalizing to bases other than ten, but encodes this information in NUMBER rather than in features. I'm skeptical at putting too much of the machinery regarding compositionality and numerals in the syntactic component itself, though, due to a lack of direct evidence for particular proposals (my own feature-checking proposal included). Moreover, conventions regarding numerical well-formedness can sometimes be flouted (discussed more below), which suggests to me that it is not (entirely) the syntactic component determining what the numerical form for a number should be. Instead, I propose that at least some of the work in ruling out particular numerals is handled by the pragmatic system.

For instance, Bogal-Allbritten (2010) proposes a neo-Gricean principle called **Avoid Synonymy** (see (36)), meant to explain the distribution of evaluativity with comparative aspect and absolute aspect marked verbs in Navajo.[6] Perhaps we might consider the use of such a pragmatic principle in generally ruling out illicit numerals, since these numerals have the same truth conditions as their competitor numerals

[6]See Rett 2007 for a similar (though unnamed) principle, and also Rett 2015 for additional discussion regarding evaluativity and markedness. A reviewer notes that this principle seems to be a manifestation of an elsewhere rule or Pāninian ordering rule, which prefers more specific rules before general rules (Kiparsky 1973, 1979).

(e.g., *thirty-one* and **twenty-eleven* have the same denotation, 31) but are marked due to violating the speaker's knowledge of the numeral paradigm.

(36) **Avoid Synonymy** (Bogal-Allbritten 2010)
Avoid a derivation producing an expression that has the same truth conditions as a competing derivation containing a less marked adjective.

More generally, we might consider the lack of forms such as *twenty-eleven* or *thirty-fourteen* as being ruled out by more general principles related to blocking, the phenomenon where marked forms are blocked by more unmarked forms. A canonical example is how the derivationally transparent but marked noun *stealer* is blocked by the lexicalized form *thief*; both have identical meanings, at least on a naive view, but the conventionalized form *thief* is preferred over the form *stealer*.

We might object that blocked forms do surface occasionally; *stealer* does have a meaning and occasionally surfaces, for instance. Blocked forms of numerals occasionally surface as well, although admittedly they are somewhat rarer. For instance, the numerical base for thousands in English can be re-expressed with a base for hundreds, provided the multiplicative numeral itself has an increase in its base (see (37)). Counting can be done incorrectly using numbers of too high a base (38a), and numerals constructed in this way can sometimes be used for humorous effect, such as when someone wants to make a comment on their age (38b).

(37) a. two thousand five hundred (=2500)
b. twenty-five hundred (=2500)

(38) a. ..., thirty-nine, thirty-ten, thirty-eleven, ...
b. Well Rob, I just turned thirty-eleven (and I do give you credit for the phraseology on that), and I saw my first silver hairs about 3 years ago. (Google)

Although how precisely blocking is to be formalized is still a matter of debate,[7] I do not think it is problematic to assume that blocking plays a role in the generation and subsequent filtering of possible numerals; it is the speaker's knowledge of the numeral paradigm that blocks illicit numerals from arising, just as it seems to be the speaker's knowledge of *thief* that blocks *stealer* from arising. I'm keen on highlighting this point here, since this sort of knowledge of the system itself seems to plausibly play a role in constraining the interpretation of indeterminate numerals.

[7] See Embick & Marantz (2008) for discussion.

4 Grammatical alternatives

In order to model the ignorance implicature that characterizes indeterminate numerals, I make use of alternative semantics. Alternatives are familiar from Hamblin's (1973) and Karttunen's (1977) work on questions. In the kind of approach they develop, the meaning of a question is a set of propositions corresponding to answers to the question; a question such as *Who left?* might be represented as in (39), a set of propositions varying on the individual who did the leaving. What the meaning of question is, then, is a set of alternatives which raise an issue as to which alternative is the true alternative.

(39) $[\![\textit{Who left?}]\!] = \{\lambda w \exists x.\mathbf{leave}_w(x) \mid \mathbf{person}(x)\}$

I take the contribution of indeterminate numerals as expressing a set of alternatives. More about these alternatives will be discussed later in this paper, but for present purposes it's enough to suppose that sentences containing indeterminate numerals are related to a set of propositions corresponding to different values for the indeterminate numeral; in other words, the alternatives for a sentence containing an indeterminate numeral are propositions that vary with respect to the number that the indeterminate numeral is (in a sense) standing in for. In expressing a set of alternatives, the speaker raises the issue of which of the alternatives holds true in the actual world. By raising multiple possibilities, the speaker implicates their ignorance, uncertainty, or indifference as to which alternative is the true alternative.

I explicitly represent the alternatives as part of the compositional semantic meaning of the sentence. The best known system that does this is that of Kratzer & Shimoyama 2002, who consider denotations to be sets of alternatives. Systems like this have been used to model not just the familiar cases of questions (Hamblin 1973), and focus (Rooth 1985), but also topichood (Büring 1997), indefinites (Alonso-Ovalle & Menéndez-Benito 2003), pronouns (Kratzer & Shimoyama 2002, Kratzer 2005), modified numerals (Coppock 2016), and scalar implicatures (Chierchia 2004).

In a Hamblinized system such as this, where alternatives are represented as part of the compositional semantics, it's necessary to have a mode of composition separate from ordinary Function Application (Heim & Kratzer 1998) that can put sets of functions together with their arguments—namely, what's necessary is to have a mode of composition where we can act like we're working with functions, but in reality be composing sets of alternatives with each other. The intuition behind this mode of composition, Pointwise Function Application, is to apply all the objects from one set of alternatives to all the objects from another set of alternatives pointwise, creating another set of alternatives. This is formalized in (40) below.

(40) **Pointwise Function Application** (based on Kratzer & Shimoyama 2002)
If α is a branching node with daughters β and γ, and $[\![\beta]\!]^{d,C} \subseteq D_\sigma$ and $[\![\gamma]\!]^{d,C} \subseteq D_{\langle\sigma,\tau\rangle}$, then $[\![\alpha]\!]^{d,C} = \{c(b) \mid b \in [\![\beta]\!]^{d,C} \wedge c \in [\![\gamma]\!]^{d,C}\}$

Singleton sets of alternatives compose in more or less the usual way; one member of the set of alternatives applies to the member of the other set. Where things get more interesting is when multiple alternatives are present. Function application applies pointwise, so that each alternative in the first set is applied to each alternative in the second. In this way, these alternatives "fan outwards" (to borrow phrasing from Coppock (2016: 472)), creating expanding sets of alternatives.

The set of alternatives generated by repeated application of the Pointwise Function Application rule is existentially closed via an existential closure operator in the tree. This operator is associated with the following rule:

(41) **Existential Closure** (adapted from Alonso-Ovalle 2006)
Where $[\![A]\!] \subseteq D_{\langle st,t\rangle}$, $[\![[\exists\, A]]\!] = \{\lambda w.\exists p\,[p \in [\![A]\!] \wedge p(w)]\}$

This system will be put to use in the following sections in order to model the indeterminacy, with indeterminacy being related to a non-singleton set of alternatives.

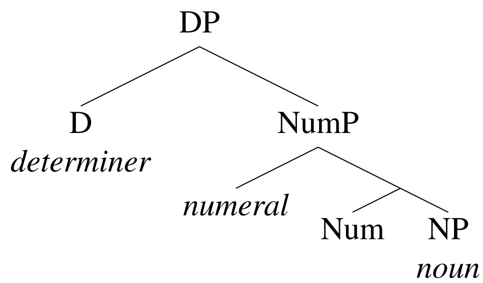
5 Semantics of ordinary numerals

First, I assume a degree semantics for cardinal numerals, following a similar move by Solt for quantity words such as *few* and *many*. Departing from Solt (2015), however, I treat simple numerals as directly denoting degrees, objects of type d. This makes a cardinal such as *twenty* have the denotation in (42). Note that this denotation has already been Hamblinized; in a non-Hamblinized system, *twenty* would simply denote the degree 20. Here, it denotes the set containing only the degree 20.

(42) $[\![\textit{twenty}]\!] = \{20\}$

Syntactically, numerals are inserted in the specifier of a NumP projection, as in (43), breaking with the syntax proposed by Ionin & Matushansky (2006) and more in line with proposals by Solt (2015) and others. NumP dominates the NP projection, but is still contained in DP. The role of Num head is to measure the cardinality of an individual (using a measure function for cardinality of individuals μ), and relate this to the denotation of the numeral in SpecNumP. How this is done is shown in (44).

(43)

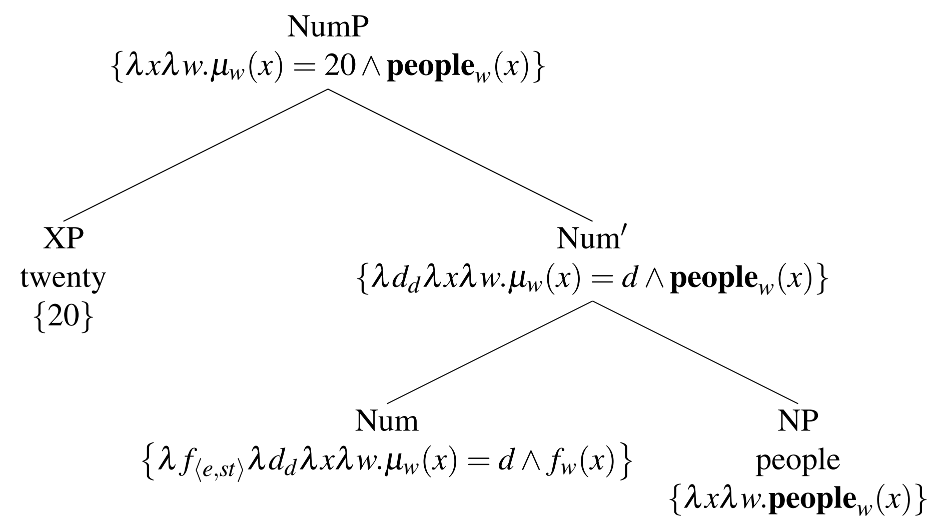


(44) $[\![\text{Num}]\!] = \left\{\lambda f_{\langle e,st\rangle}\lambda d_d \lambda x \lambda w.\mu_w(x) = d \wedge f_w(x)\right\}$

Putting these pieces together, the derivation for *twenty people* would look as in (46).[8] Num takes the NP headed by the lexical noun as an argument, and their denotations compose via the Pointwise Function Application rule. This merges with the numeral, and the numeral saturates the degree argument of Num, resulting in the singleton set containing the intensional property of being a plurality of people who measure twenty.

(45) $[\![\textit{people}]\!] = \{\lambda x \lambda w.\mathbf{people}_w(x)\}$

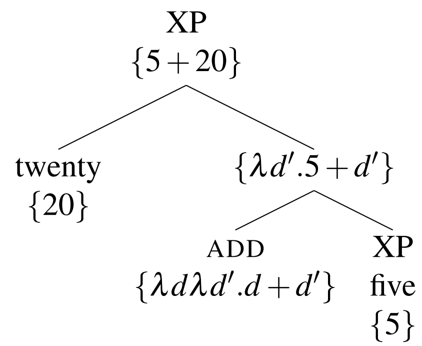
(46)



A complex, but non-indeterminate numeral can be given a similar analysis. First, the numeral is composed using ADD.

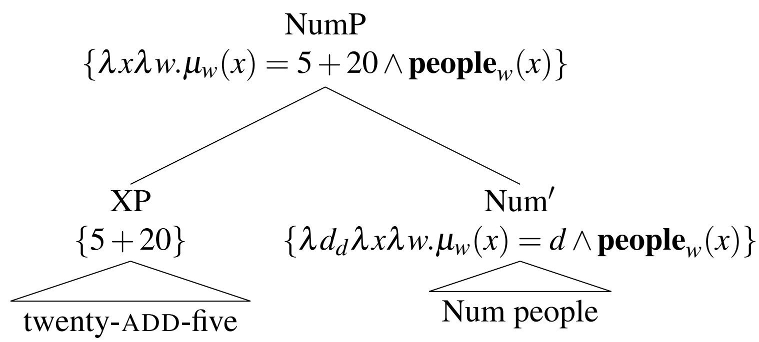
[8] It might be the case that *twenty* can be syntactically decomposed into *two* and *-ty*. This additional detail doesn't play a role in this paper, though see Mendia 2018 for discussion of multiplicative numerals with assumptions that are compatible with mine.

(47)



This numeral can then compose with its sister, an intermediate projection of NumP, via Pointwise Function Application.

(48)



In this way, no type-shift is necessary to get numerals to be an argument of Num. Numerals simply are names for degrees, and thus can directly serve as arguments to Num.

6 Analysis of indeterminate numerals

6.1 Previous analysis: Anderson (2015)

Anderson (2015) provides an analysis of English indeterminate numerals with *some*. In this analysis, the *some* element in the numeral merges with a phonologically null noun NUMBER. Similarly, the *some* NUMBER constituent combines with an additive numeral using a covert coordinate element ADD. However, an important difference is that, due to a typeclash between the semantics of Num and the indeterminate numeral, they must be lifted to the type of a generalized quantifier over degrees ($\langle dt,t \rangle$) and must raise out of the DP via quantifier raising. Schematically, this is shown in (49), where the indeterminate numeral raises to the left edge of TP and leaves behind a trace of type d as in the familiar Heim & Kratzer (1998) mode of analysis, suitably extended to type d. QR is necessary in order to fix the typeclash generated when an epistemic numeral is used; as Anderson argues, Partee's (1987)

BE type-shift is not available with indeterminate numerals due to the numeral not being expressible as a singleton: these numerals must be represented as a set of degrees. In order to have the indeterminate numeral be type-compatible with its sister, the indeterminate numeral needs to be lifted to the type of a generalized quantifier over degrees, $\langle dt, t\rangle$, and then undergo quantifier raising.

(49) [*twenty-some*$_i$ λ_i [... [t_i [Num NP]]]]

However, this analysis has two problems. First, the DP itself is an island to movement, via familiar constraints on extraction out of definite DPs. Moreover, left-branch extraction of numerals is not generally permissible in English, and so it is suspicious that this construction would allow movement of the numeral, even if it is at LF; extraction of a numeral seems possible only when it pied-pipes the NP it counts over, as seen in (50).

(50) a. *[How many]$_i$ did John see [t_i dogs]? (John saw fifteen dogs.)
b. [How many dogs]$_i$ did John see t_i?

Additionally, the analysis also runs aground due to what Bhatt & Pancheva (2004) call the Heim-Kennedy Constraint.[9] The Heim-Kennedy constraint is based on the observation that DegPs do not take scope over QPs. Bhatt & Pancheva (2004) suggest that is should be considered as a constraint on degree abstraction, and not simply DegP, as schematized in (52). Given this formulation, the analysis in Anderson (2015) would violate the constraint, as quantifier raising of indeterminate numerals out of the DP involves degree abstraction.[10]

(51) **Heim-Kennedy Constraint** (as cited in Bhatt & Pancheva 2004: 15)
If the scope of a quantificational DP contains the trace of a DegP, it also contains that DegP itself.

(52) $*\lambda d \ldots \text{QP} \ldots d \ldots$ (Bhatt & Pancheva 2004: (25))

Taken together, these problems point in a different direction for the analysis of indeterminate numerals. An analysis where numerals are properties of degrees and must QR out of the DP cannot be correct, due to it violating several well-known constraints on movement in English. The analysis of indeterminate numerals I build in the following sections solves this problem.

[9] I thank Nicholas Fleisher (p.c.) for suggesting this line of thought.

[10] This also relies on an assumption that a QP takes scope over material within the QP.

6.2 The meaning of *some*

Based on the parallels that *some* shows with Spanish *algún*, I propose treating *some* in a similar way, in particular supposing that *some* triggers minimal domain widening via an anti-singleton constraint. This follows a proposal by Weir (2012), who analyzes the determiner *some* as making use of a subset selection function f that is constrained to have a non-singleton co-domain. In this way, *some* (like *algún*) can generate an implicature that the speaker cannot (or will not) narrow the domain to a single alternative, modeling the epistemic effect.

(53) $[\![some]\!] = \lambda f_{\langle et,et\rangle}\lambda P\lambda Q\colon$ **anti-singleton**$(f).\exists x\,[f(P)(x) \wedge Q(x)]$ (Weir 2012: (14))

Where I will depart from this analysis is in treating *some* as a quantificational determiner. Rather, building on previous work in alternative semantics (Kratzer & Shimoyama 2002, Szabolcsi 2015), I consider *some* to actually signal the presence of two operations. The first is an existential operator $\exists$ at the clause level that provides existential closure over alternatives (flattening them into a single proposition). This operator has been mentioned already, as (41), repeated below. An analysis of *some* as a quantificational determiner, as in Anderson (2015) and Weir (2012), requires that *some* introduce existential quantification. In a fully Hamblinized semantic system like the one I develop here, indefinites introduce sets of alternatives (see also Alonso-Ovalle & Menéndez-Benito (2003), AnderBois (2011)), with the existential closure operator performing the role that existential quantification provided in the determiner *some*.

(41) **Existential Closure** (adapted from Alonso-Ovalle 2006)
Where $[\![A]\!] \subseteq D_{\langle st,t\rangle}$, $[\![[\exists\, A]]\!] = \{\lambda w.\exists p\,[p \in [\![A]\!] \wedge p(w)]\}$

The second operation is marked by a morpheme ANTI-SINGLETON (abbreviated in trees as A-S). It has the role of ensuring that the alternatives generated by the constituent sister to ANTI-SINGLETON are a non-singleton set of alternatives. This mirrors in some respects the anti-singleton presupposition in Alonso-Ovalle & Menéndez-Benito's (2010) discussion of Spanish *algún*; where the anti-singleton presupposition in *algún* restricts the subset selection function in *algún* to having a non-singleton co-domain, ANTI-SINGLETON ensures that the set of alternatives generated at the point in the tree where ANTI-SINGLETON is merged are not a singleton. In English, ANTI-SINGLETON is spelled out as *some*.

To clarify this point, *some* is the spell-out of ANTI-SINGLETON in a particular syntactic configuration. When ANTI-SINGLETON is c-commanded by $\exists$, the phonological form for *some* is inserted at the syntactic position where ANTI-SINGLETON has been merged. This entails a realization theory of morphology, such as Distributed Morphology (Marantz 1997). What is most important to take away from this discussion is that the semantic work of *some*, the introduction of alternatives and the subsequent existential closure of them, is syntactically represented in two positions, with the phonological form for *some* being realized in the position of ANTI-SINGLETON.

Returning to the semantics of ANTI-SINGLETON, ANTI-SINGLETON is somewhat unusual: Because it needs access to the set of alternatives itself, rather than the individual alternatives within the set, it needs to exist in some sense "outside" of the normal composition rule in the semantic system I'm assuming, Pointwise Function Application. Instead, it combines with its argument via ordinary function application.[11] ANTI-SINGLETON takes a set of alternatives, presupposes that a contextually defined subset selection function yields a non-singleton subset of the set of alternatives, and then passes that subset up the tree for computation. This is given in (54).

(54) $[\![\text{ANTI-SINGLETON}]\!] = \lambda p_{\langle\sigma,t\rangle} : \textbf{anti-singleton}(f).f(p)$

What makes ANTI-SINGLETON important in this analysis is that, by using it, the speaker signals that they are forcing the semantic derivation to include at least two alternatives, and hence signaling that there are multiple epistemic possibilities at issue. In this way, by forcing multiple alternatives, the speaker can generate the implicature that they are ignorant towards which possibility is the true possibility in the world of evaluation.

7 An alternative semantics for indeterminate numerals

7.1 Upper-boundedness as implicature

In Anderson 2015, I speculated that the upper-boundedness of indeterminate numerals comes from competition with other, larger numerals, and that an implicature can be used to derive the upper-boundedness. Anderson (2016) suggests that the upper-bound can be given syntactically through the use of syntactic features encoding the next lower base, although the strategy is also not fully fleshed out. In this approach, *twenty* would check a base feature on its sister that indexes it with base 10^0. In a

[11] As a reviewer points out, there is a tension here in that this analysis needs both the usual Function Application rule as well as Pointwise Function Application. Moreover, this representation format also muddies the distinction between sets of alternatives and characteristic sets. Moving to a compositional system such as that proposed by Charlow (2014, 2019) might make resolve this situation, as a reviewer suggests. I'm sympathetic to such a move, but postpone the question of if and how my present analysis can be formulated with assumptions closer to Charlow's for another time.

similar move, Mendia (2018) proposes to fix the denotation of NUMBER in such a way so as to give it precisely the correct base for its position within the syntactic structure.

Here, I want to return to the original intuition that it is the form of the numeral that is fixing the upper-boundedness, once again deriving it as an implicature. The benefit of this approach is that the upper-bounded constraint on the indeterminate numeral can be constructed compositionally using familiar tools from the analysis of scalar implicatures, rather than treated as a constraint on the construction as a whole. The essential idea will be to treat the upper-bounded interpretation as a kind of Q (quantity) implicature, an implicature generated by flouting Grice's Maxim of Quantity, the communicative principle that a speaker ought to be as informative as possible (Horn 1984, Grice 1957). In modifying one numeral rather than another, the speaker generates the inference, via the existence of a scale, that they could not commit themselves to the information content carried by members up the scale.

First, it is necessary to show that the lower-boundedness of indeterminate numerals is not an implicature, but is part of the asserted meaning of the sentence. This can be done by forcing a contradiction, as in (55).

(55) *There were thirty-some people at the party, and there weren't even thirty. (contradiction)

Next, (56) shows that the lower-bounded meaning component cannot be reinforced; these sentences sound redundant. This again suggests that it is part of the asserted meaning of the sentence. This is not surprising, I think, but showing this is necessary in order to make a distinction between the asserted and implicated components of indeterminate numerals.[12]

(56) a. ??There were thirty-some people at the party, definitely at least thirty.
b. ??I have a hundred-some stamps in my collection, definitely at least one hundred.

In comparison, the upper-bound of indeterminate numerals does seem to be reinforceable.

(57) a. There were thirty-some people at the party, definitely not more than forty.

[12]There may be some ways of rescuing the sentences in (56), such as if the lower-bound is contextually relevant and the adverb *definitely* taken to be emphasizing meeting that bound. But, in out of the blue contexts, the preferred interpretation of (56) is one where *definitely* contributes information already asserted in the sentence, giving a sense of redundancy. I thank Ai Taniguchi (p.c.) for the observation that there is a context where these sentences are acceptable.

b. I have a hundred-some stamps in my collection, definitely less than two hundred.

That the upper-bound is reinforceable, or at least more easily reinforceable than the lower-bound, suggests that it is not asserted as part of the conventional meaning of the indeterminate numeral; rather, the fact that it can be independently asserted and reinforced suggests that it behaves more like inferred meaning, e.g. an implicature.

How is this implicature calculated? Minimally, we need to consider what the competitors to a numeral like *twenty* are. What we notice is that for *twenty-some*, of course, the upper bound is set by 30; for *two hundred-some*, the upper bound is set by 300. The competitors for a numeral involved in an indeterminate numeral, at least for English indeterminate numerals formed from *some*, are formed by abstracting over the multiplier for the largest base in the numeral. The immediately relevant alternative is the next higher multiplier for the base. For *twenty* (2×10), the relevant alternative for the implicature is calculated by picking the next highest multiplier (3×10). And, perhaps more generally, the upper bound can be related to the matter of blocking of illicit numerals (see 3.4). A speaker has knowledge of the numeral paradigm in their language, and this can be used to set an upper bound for an indeterminate numeral, in order to rule out truth-conditionally equivalent expressions that (if they were uttered) would not fit in the numeral paradigm. This seems to be easier said than done, but it may very well be that the same principles that rule out illicit numerals also play a role in constraining indeterminate numerals.

7.2 Computation of indeterminate numerals

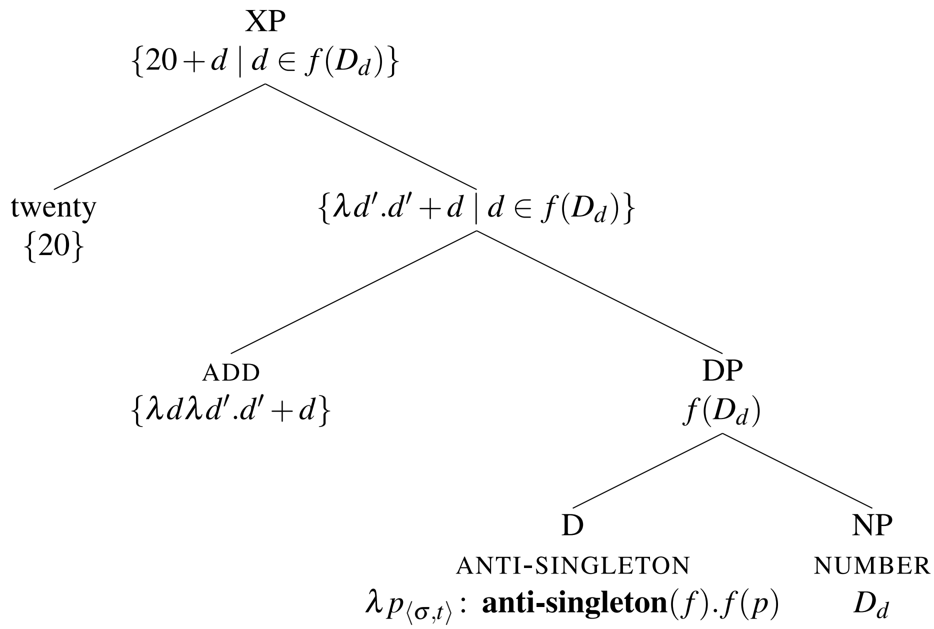
The computation for an indeterminate numeral proceeds in largely the same fashion as for an ordinary numeral; both indeterminate numerals and ordinary numerals will be typed as degrees d, making them simply arguments to a Num head that mediates between the lexical NP and the numeral in SpecNumP. Where indeterminate numerals differ from ordinary numerals is in introducing into the derivation a non-singleton set of alternatives.

To begin, we'll consider the indeterminate numeral itself. First, ANTI-SINGLETON combines with NUMBER. I assume a weak semantics for NUMBER: it simply denotes the domain of degrees, D_d.[13] When ANTI-SINGLETON combines with NUMBER, it

[13]It seems quite difficult for the indeterminate numeral to denote a fractional number, such as *twenty-some* denoting 25.5. If *some* NUMBER is just simply denoting the domain of degrees, it's somewhat unclear why this should be, given that some authors (e.g., Fox & Hackl (2007), for example) assume that the domain of degrees is a subset of the real numbers $\mathbb{R}$, and not of the integers. There's two options that come to mind here. One possibility would be to have NUMBER denote in the integers $\mathbb{Z}$ or in the natural numbers $\mathbb{N}$. A second possibility would be to have additional entailments stemming from a more general semantics of numerals that numerals necessarily count

selects a subset of NUMBER. This set is guaranteed to have at least two members. For an indeterminate numeral such as *twenty-some*, the numeral will denote a set of degrees of the form $20+d$, where d is a member of D_d, and this set will have at least two members. This derivation for *twenty-some* is given in (58).

(58)



Next, Num′ is applied pointwise to the denotation of the numeral; for each degree in the denotation of the numeral, Num′ applies to it, saturating Num's degree argument. This allows the alternatives from the numeral to continue to fan outwards throughout the course of the semantic derivation.

atomic individuals. An atomicity constraint of this type would then force *some* NUMBER to always denote an integer. I have very little else to say about these possibilities here, though, and leave the question for further research.

(59)

- NumP $\{\lambda x \lambda w.\mu_w(x) = 20 + d \wedge \textbf{people}_w(x) \mid d \in f(D_d)\}$
  - XP $\{20 + d \mid d \in f(D_d)\}$
    - twenty [ADD A-S NUM]
  - Num′ $\{\lambda d_d \lambda x \lambda w.\mu_w(x) = d \wedge \textbf{people}_w(x)\}$
    - Num $\{\lambda f_{\langle e,st\rangle} \lambda d_d \lambda x \lambda w.\mu_w(x) = d \wedge f_w(x)\}$
    - NP people $\{\lambda x \lambda w.\textbf{people}_w(x)\}$

As argued for previously, the upper-bounded inference should be treated as an implicature. (As we can already see, it does not come directly from the semantics of the indeterminate numeral.) I assume that (for this example), at the level of NumP, an implicature is calculated based on competitors to the modified numeral. The relevant alternatives for the quantity implicature are given in (60b), where the next numeral "up" from the modified numeral, based on increasing the value of the multiplier in the numeral, sets an upper-bound for the measure function over individuals μ (see also section 7.1). The NumP and the implicature can be intersected to give the strengthened, upper-bounded interpretation in (61).

(60) a. NumP: $\{\lambda x \lambda w.\mu_w(x) = 20 + d \wedge \textbf{people}_w(x) \mid d \in f(D_d)\}$
b. Implicature: $\{\lambda x \lambda w.\mu_w(x) < 30 \wedge \textbf{people}_w(x)\}$

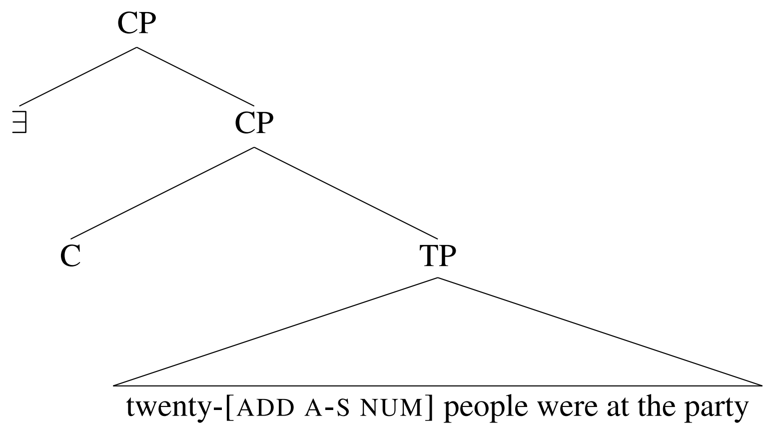
(61) Strengthened: $\{\lambda x \lambda w.\mu_w(x) = 20 + d \wedge \mu_w(x) < 30 \wedge \textbf{people}_w(x) \mid d \in f(D_d)\}$

Setting the upper-bound in this way ensures that, no matter what the subset selected from D_d is, the addition of any member of that subset with the modified numeral will never be larger than the competitor. A schematization of these alternatives is given in (62).

(62)
$$\left\{\begin{array}{c} \lambda x \lambda w.\mu_w(x) = 20 + 1 \wedge \mu_w(x) < 30 \wedge \textbf{people}_w(x), \\ \lambda x \lambda w.\mu_w(x) = 20 + 2 \wedge \mu_w(x) < 30 \wedge \textbf{people}_w(x), \\ \dots \\ \lambda x \lambda w.\mu_w(x) = 20 + 8 \wedge \mu_w(x) < 30 \wedge \textbf{people}_w(x), \\ \lambda x \lambda w.\mu_w(x) = 20 + 9 \wedge \mu_w(x) < 30 \wedge \textbf{people}_w(x) \end{array}\right\}$$

These alternatives percolate upward through the derivation, until arriving at the $\exists$ operator, which I will suppose is adjoined to CP. To recapitulate, the role of $\exists$ is to transform the set of alternatives it combines with into a singleton. To do this, it takes the alternatives which have been created in the course of the derivation, the set of epistemic possibilities, and asserts that one of them holds in the world of evaluation.

(63)



(64) $[\![\exists$ *twenty*-[ADD A-S NUM] *people were at the party*$]\!]$
$= \{\lambda w.\exists p\,[p \in [\![$*twenty*-[ADD A-S NUM] *people were at the party*$]\!] \wedge p(w)]\}$

In this way, indeterminate numerals generate a non-singleton set of alternatives, with the upper-bound given as a quantity implicature.

It's worth stressing at this point that it is not the values of NUMBER (or rather, *some* NUMBER) that are constrained in the course of the derivation for the indeterminate numeral. NUMBER itself is quite weak, simply the domain of degrees, and the value of *some* NUMERAL is a non-singleton subset of the domain of degrees. But, the value of *some* NUMERAL is not directly constrained by the other material in the numeral; there is no selection mechanism or anaphoric connection between *some* NUMERAL and the rest of the numeral. Rather, it is addition of the implicature that rules out certain alternatives. In the case of *twenty-some*, for instance, the "less than thirty" implicature causes any alternatives with a numerical value (computed by the addition of 20 plus members of the relevant non-singleton subset of the domain of degrees D_d) greater than 30 to be false.

How does this story fare with numerals other than *twenty-some*? The intuitions seem to be as expected from the analysis, though the picture for calculation of the upper-bounded implicature is somewhat complicated. Take the numerals in (65), along with their bracketings in (66). *Two million-some* has the upper-bound we might expect, 3 million, given its syntax. This is arrived at by generating an implicature based on increasing the multiplier for the base *million* from 2 to 3. *Six hundred thousand*'s upper-bound is generated by comparison with *seven hundred thousand*, and the upper-bounded implicature generated for *two million five hundred thousand-some* is *two million six hundred thousand*, with *six hundred thousand* being in competition with *five hundred thousand*, a move made by increasing the multiplier for 100 from 5 to 6.

(65) a. two million-some (2 million, 3 million)
b. six hundred thousand-some (600000, 700000)
c. two million 500 thousand-some (2.5 million, 2.6 million)

(66) a. [[two × million] ADD *some* NUMBER]
b. [[[six × hundred] × thousand] ADD *some* NUMBER]
c. [[two × million] ADD [[[five × hundred] × thousand] ADD *some* NUMBER]]

This is not to say that the calculation of the implicature is trivial; there are a number of questions that remain about how the upper-bounded implicature is calculated, not all of which can be answered at this point.[14] For instance, why is it the multiplier (and not the base) that is involved in generating the implicature? The intuitive answer seems to be related to what numbers or numerals are contrastive with what has been spoken, with numerals of the same base but different multipliers generally being the most contrastive (e.g., *thirty* and *forty* form a more salient pair than *thirty* and *three hundred*), but this is admittedly not totally satisfying, nor straightforward to cash out formally. There is also the issue of how deep into the hierarchical syntax the mechanism for generating the upper-bounded implicature can see; *two* in (66a) is more embedded than *six* in (66b). And finally, the relevance of structure also feeds into the question of when the implicatures are calculated. For (65c), the implicature must be calculated internally to the numeral, due to the multiplier for the highest base not forming part of the implicature generation; the implicature is generated over *five hundred thousand*, rather than *two million*. Clearly, much work needs to be done in order to understand the semantics of indeterminate numerals more fully.

7.3 Verbs of saying and indeterminate numerals

The analysis developed in the previous sections relies on a syntactically represented $\exists$ operator. This operator is adjoined to CP and scopes over all the alternatives, serving to close off the set of alternatives, e.g. to transform it into a set containing a single proposition. The effect of this is to anchor the ignorance implicature to the speaker, since it is the speaker who utters this proposition.

A quirk of this analysis is that closure of the alternatives generated by *some* could potentially occur at different syntactic levels; there is no principle that requires that $\exists$ be adjoined to the matrix CP. Put another way, the analysis predicts that closure

[14] I thank a reviewer for pressing me on some of these issues, and I regret that I'm not able to completely flesh out these details yet.

of the alternatives could occur in (at least) two positions in a sentence with a finite embedded clause: at the highest CP, or at the CP corresponding to the embedded clause. These possibilities are schematized in (67), where (∃) marks a possible position for existential closure.

(67) $[_{CP}$ (∃) C $[_{TP}$ ... $[_{VP}$ V $[_{CP}$ (∃) C ...

With a verb that encodes an attitude towards a proposition, such as a verb of saying, this raises the possibility for an interpretational difference, depending on the height of ∃. Above the verb (at the highest CP), the ignorance will be anchored to the speaker, as in the cases discussed in previous sections. Under the verb, at the embedded CP, the ignorance will instead be anchored to the sentential subject. Thus, in a sentence with *some* in an embedded clause and under a verb of saying, we predict an ambiguity in which individual the ignorance implicature is anchored to.

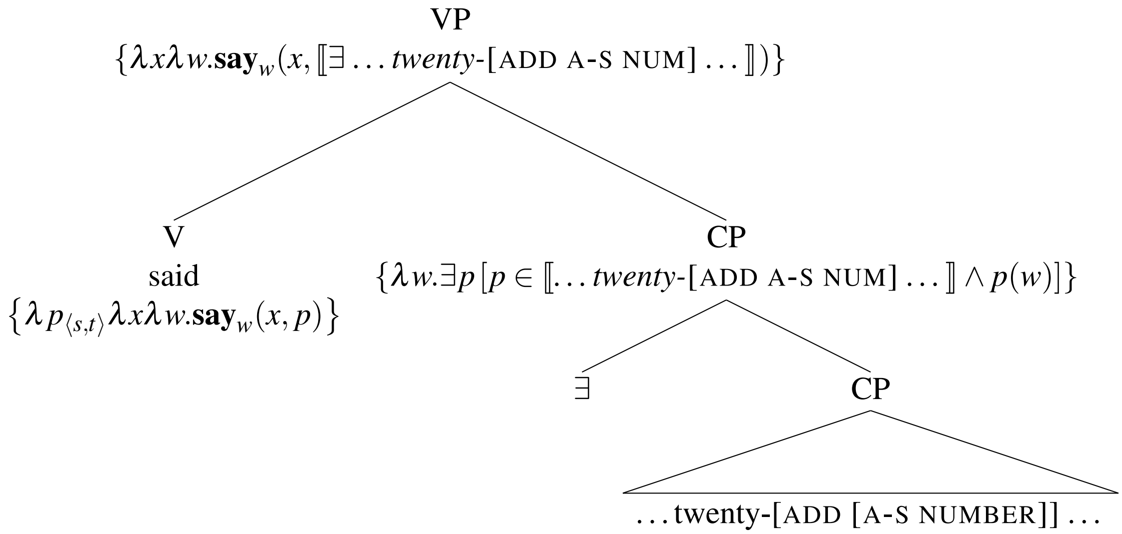
This is what we find, as shown with example (68). This sentence is two-ways ambiguous, with ignorance being expressed by the sentential subject, the subject of the saying, or by the speaker; the paraphrases in (68a) and (68b) demonstrate this intuition, while (69) shows this via follow-ups that directly assert ignorance on behalf of either the speaker or subject. Thus, it seems that the analysis correctly predicts that sentences with an indeterminate numeral in the scope of an attitude verb are ambiguous.

(68) John said that there were twenty-some people were at the party.
  a. John said how many people were at the party, but I don't know precisely what he said. (speaker ignorance)
  b. John said something and expressed ignorance as to the precise number of people at the party. (subject ignorance)

(69) John said that there were twenty-some people at the party...
  a. ... but I don't know exactly how many (he said there were). (speaker ignorance)
  b. ... but he didn't know exactly how many. (subject ignorance)

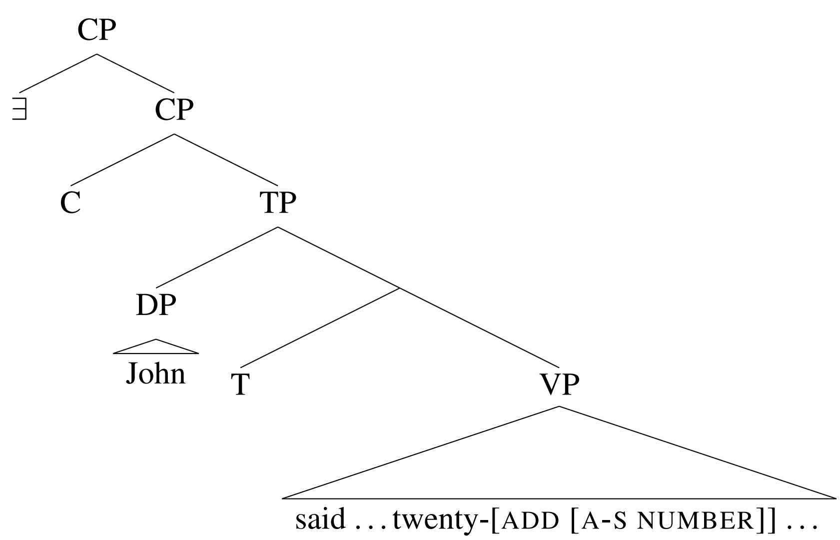
The anchoring of ignorance to different individuals can be essentially thought of as a relatively ordinary scope ambiguity; the relative scope of the ∃ operator with respect to the attitude verb determines when the set of propositional alternatives is flattened to a single proposition. If the operator scopes below the attitude verb, the alternatives are flattened into a single proposition, which is the proposition expressed by the one doing the saying.

(70)



On the other hand, if the ∃ operator scopes above the verb, as in (71), then the alternatives from the indeterminate numeral persist to the top of the clause. Due to the Pointwise Function Application rule, the verbal meaning is factored into the alternatives generated by the indeterminate numeral. The resulting set of alternatives is a set of alternatives that vary by which proposition was said. By allowing the alternatives to persist past the level of the verb, what is constructed is a set of alternatives that express possible propositions that were said, as in (72). Effectively, this creates ignorance about which particular proposition was uttered by someone, but only commits the speaker of the root clause to ignorance, not the person who said (the content of) the embedded clause. The logical form for this is provided in (72).

(71)



(72) $\{\lambda w.\mathbf{say}_w(\mathbf{john}, \lambda w \exists x \exists d.\mu_w(x) = 20 + d \wedge \mu_w(x) < 30 \wedge \ldots) \mid d \in f(D_d)\}$

To briefly summarize, the analysis of indeterminate numerals predicted that sentences with an indeterminate numeral in a clause embedded under an attitude verb such as *say* should be ambiguous, due to the set of alternatives being able to be closed off at the level of either the embedded CP or the matrix CP. This prediction was borne out, providing evidence for the claim that closure of the alternatives is contributed by a separate existential operator that can be variously merged at the level of either the matrix CP or the embedded CP.

8 Conclusion

This chapter investigated the use of *some* in forming approximate, uncertain meanings with numerals, what I call indeterminate numerals. These numerals have the structure and semantics of ordinary numerals (degree-denoting expressions), but are special in that they make use of a morpheme ANTI-SINGLETON (spelled out as *-some*) that forces the generation of at least two alternatives. The generation of multiple alternatives models the uncertainty inherent to these numerals. These numerals gain an additional upper-bounded inference via a second quantity implicature, based on the value of the numeral that *-some* attaches to. This work shows how morphemes associated with quantificational elements such as indefinite determiners can interact with degrees when placed in certain syntactic configurations, and sheds light on the quantificational mechanisms used in computing ignorance over sets of degrees.

References

Alonso-Ovalle, Luis. 2006. *Disjunction in alternative semantics*: University of Massachusetts Amherst dissertation.

Alonso-Ovalle, Luis & Paula Menéndez-Benito. 2003. Some epistemic indefinites. In Shigeto Kawahara & Makoto Kadowaki (eds.), *Proceedings of the North East Linguistics Society (NELS) 33*, 1–12. Amherst, MA: GLSA.

Alonso-Ovalle, Luis & Paula Menéndez-Benito. 2010. Modal indefinites. *Natural Language Semantics* 18. 1–31.

Alonso-Ovalle, Luis & Paula Menéndez-Benito. 2013. Two views on epistemic indefinites. *Language and Linguistics Compass* 7. 105–122.

AnderBois, Scott. 2011. Sluicing as anaphora to issues. In Nan Li & David Lutz (eds.), *Semantics and Linguistic Theory 20*, 451–470.

Anderson, Curt. 2014. Approximation of complex numerals using *some*. In Claire Renaud, Carla Ghanem, Verónica González López & Kathryn Pruitt (eds.), *Proceedings of the Western Conference on Linguistics (WECOL) 2013*, 131–143.

Anderson, Curt. 2015. Numerical approximation using *some*. In Eva Csipak & Hedde Zeijlstra (eds.), *Proceedings of Sinn und Bedeutung 19*, 54–69.

Anderson, Curt. 2016. *Intensification and attenuation across categories*: Michigan State University dissertation.

Barwise, John & Robin Cooper. 1981. Generalized quantifiers and natural language. *Linguistics and Philosophy* 4(2). 159–219.

Bhatt, Rajesh & Roumyana Pancheva. 2004. Late merger of degree clauses. *Linguistic Inquiry* 35(1). 1–45.

Bogal-Allbritten, Elizabeth. 2010. Positively uninformative. In Michael Yoshitaka Erlewine & Yasutada Sudo (eds.), *Proceedings of the MIT workshop on comparatives 2010*, 19–34.

Büring, Daniel. 1997. *The meaning of topic and focus: the 59th Street Bridge accent*. London: Routledge.

Charlow, Simon. 2014. *On the semantics of exceptional scope*: New York University dissertation.

Charlow, Simon. 2019. The scope of alternatives: indefiniteness and islands. *Linguistics and Philosophy* .

Chierchia, Gennaro. 2004. Scalar implicatures, polarity phenomena, and the syntax/pragmatics interface. In Adriana Belletti (ed.), *Structures and beyond*, vol. 3, 39–103. Oxford University Press.

Condoravdi, Cleo. 2015. Ignorance, indifference, and individuation with wh-ever. In Luis Alonso-Ovalle & P. Menéndez-Benito (eds.), *Epistemic indefinites: Exploring modality beyond the verbal domain*, 213–243. Oxford University Press.

Coppock, Elizabeth. 2016. Superlative modifiers as modified superlatives. In Mary Moroney, Carol-Rose Little, Jacob Collard & Dan Burgdorf (eds.), *Semantics and Linguistic Theory 26*, 471–488.

Danon, Gabi. 2012. Two structures for numeral-noun constructions. *Lingua* 122(12). 1282–1307.

Embick, David & Alec Marantz. 2008. Architecture and blocking. *Linguistic Inquiry* 39(1). 1–53.

Fox, Danny & Martin Hackl. 2007. The universal density of measurement. *Linguistics and Philosophy* 29(5). 537–586.

Geurts, Bart. 2006. Take five: The meaning and use of a number word. In Svetlana Vogeleer & Liliane Tasmowski (eds.), *Non-definiteness and plurality*, 311–329. John Benjamins Publishing Company.

Grice, H. Paul. 1957. Meaning. *The philosophical review* 377–388.

Hamblin, Charles. 1973. Questions in Montague English. *Foundations of Language* 10(1). 41–53.

Haspelmath, Martin. 1997. *Indefinite pronouns*. Oxford University Press.

Heim, Irene & Angelika Kratzer. 1998. *Semantics in generative grammar*. Wiley-Blackwell.

Hofweber, Thomas. 2005. Number determiners, numbers, and arithmetic. *The Philosophical Review* 114(2). 179–225.

Horn, Laurence. 1984. Toward a new taxonomy for pragmatic inference: Q-based and R-based implicature. In Deborah Schiffrin (ed.), *Meaning, form, and use in context: Linguistic applications*, Georgetown University Press.

Hurford, James R. 1975. *The linguistic theory of numerals*. Cambridge: Cambridge University Press.

Ionin, Tania & Ora Matushansky. 2006. The composition of complex cardinals. *Journal of Semantics* 23. 315–360.

Karttunen, Lauri. 1977. Syntax and semantics of questions. *Linguistics and Philosophy* 1(1). 3–44.

Kayne, Richard S. 2005. A note on the syntax of quantity in English. In Richard S. Kayne (ed.), *Movement and silence*, 176–214. Oxford: Oxford University Press.

Kennedy, Christopher. 2015. A "de-Fregean" semantics (and neo-Gricean pragmatics) for modified and unmodified numerals. *Semantics and Pragmatics* 8(10). 1–44. http://dx.doi.org/10.3765/sp.8.10.

Kiparsky, Paul. 1973. 'Elsewhere' in phonology. In Stephen Anderson & Paul Kiparsky (eds.), *A festschrift for Morris Halle*, 93–106. Holt, Rinehart and Winston.

Kiparsky, Paul. 1979. *Pānini as a variationist*. Cambridge, MA: MIT Press.

Kratzer, Angelika. 2005. Indefinites and the operators they depend on: From Japanese to Salish. In Gregory Carlson & Francis J. Pelletier (eds.), *Reference and quantification: The Partee effect*, CSLI Publications.

Kratzer, Angelika & Junko Shimoyama. 2002. Indeterminate pronouns: The view from Japanese. In Yukio Otsu (ed.), *Third Tokyo Conference on Psycholinguistics*, 1–25. Tokyo: Hituzi Syobo.

Landman, Fred. 2003. Predicate-argument mismatches and the adjectival theory of indefinites. In Martine Coene & Yves D'hulst (eds.), *From NP to DP: The syntax and semantics of noun phrases*, vol. 1, 211–237. Amsterdam: John Benjamins Publishing Company.

Lasersohn, Peter. 1999. Pragmatic halos. *Language* 75(3). 522–551.

Marantz, Alec. 1997. No escape from syntax: Don't try morphological analysis in the privacy of your own lexicon. *University of Pennsylvania Working Papers in Linguistics* 4(2). 201–225.

Mendia, Jon Ander. 2018. Epistemic numerals. In Sireemas Maspong, Brynhildur Stefánsdóttir, Katherine Blake & Forrest Davis (eds.), *Semantics and Linguistic Theory 28*, 493–511.

Morzycki, Marcin. 2016. *Modification*. Cambridge University Press.

Nouwen, Rick. 2010. Two kinds of modified numerals. *Semantics and Pragmatics* 3. 1–41.

Partee, Barbara. 1987. Noun phrase interpretation and type-shifting principles. In Jeroen Groenendijk, Dick de Jongh & Martin Stokhof (eds.), *Studies in Discourse Representation Theory and the theory of generalized quantifiers*, 115–143. Foris Publications.

Rett, Jessica. 2007. Antonymy and evaluativity. In Tova Friedman & Masayuki Gibson (eds.), *Semantics and Linguistic Theory 17*, 210–227.

Rett, Jessica. 2015. *The semantics of evaluativity*. Oxford University Press.

Rooth, Mats. 1985. *Association with focus*: University of Massachusetts PhD Thesis.

Rothstein, Susan. 2013. A fregean semantics for number words. In Maria Aloni, Michael Franke & Floris Roelofsen (eds.), *19th Amsterdam Colloquium*, 179–186.

Sauerland, Uli & Penka Stateva. 2007. Scalar vs. epistemic vagueness: Evidence from approximators. In Tova Friedman & Masayuki Gibson (eds.), *Semantics and Linguistic Theory 17*, 228–245.

Solt, Stephanie. 2015. Q-adjectives and the semantics of quantity. *Journal of Semantics* 32. 221–273.

Stevens, Jon & Stephanie Solt. 2018. The semantics and pragmatics of "some 27 arrests". In Ava Irani & Milena Šereikaitė (eds.), *University of Pennsylvania working papers in linguistics*, 179–188.

Strawson, P. F. 1974. *Subject and predicate in logic and grammar*. London: Methuen.

Szabolcsi, Anna. 2010. *Quantification*. Cambridge University Press.

Szabolcsi, Anna. 2015. What do quantifier particles do? *Linguistics and Philosophy* 38(2). 159–204.

Weir, Andrew. 2012. *Some*, speaker knowledge, and subkinds. In Rasmus K. Rendsvig & Sophia Katrenko (eds.), *Proceedings of the ESSLLI 2012 student session*, 180–190.

Zweig, Eytan. 2005. Nouns and adjectives in numeral NPs. In Leah Bateman & Cherlon Ussery (eds.), *Proceedings of the North East Linguistics Society 35*, 663–675.

Curt Anderson
Collaborative Research Center 991
Heinrich-Heine-Universität Düsseldorf
Universitätsstraße 1
40225 Düsseldorf, Germany
andersc@hhu.de